PROBLEM OF THE MECHANICAL CHARACTERISTICS OF SOILS WITH ALLOWANCE FOR THEIR VISCOPLASTIC PROPERTIES UNDER A CYCLIC DYNAMIC LOADING

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Narozhnaya and Rykov [1] and Rykov and Skobeev [2] describe a method for and the results of determination of the mechanical characteristics of soils with allowance for their viscoplastic properties under short-term dynamic loads. The actual quantitative characteristics are adequately substantiated only as applies to a one-time loading.

The results of determination of the mechanical characteristics of sandy and clayey soils with allowance for their cyclic loading are presented below. The introduction of refinements to the formulation of the soil model was required in this case [1, 2]. Moreover, new data on limiting dynamic diagrams [3], and also additional experimental results for a cyclic dynamic loading [4] are defined more precisely for these soils.

It is assumed that the compressibility of a soil in uniaxial dynamic compression is described, as in [1, 2], by a strain of the form [5]

$$\partial\varepsilon/\partial t = (1/E(\varepsilon, \varepsilon_*))\,\partial\sigma/\partial t + g(\sigma, \varepsilon),\; E(\varepsilon, \varepsilon_*) = \begin{cases} E(\varepsilon), & \sigma > f(\varepsilon, \varepsilon_*), \\ E_*(\varepsilon, \varepsilon_*) & \sigma \leqslant f(\varepsilon, \varepsilon_*). \end{cases} \tag{1}$$

Here, $g(\sigma, \varepsilon)$, $E(\varepsilon)$, and $E_*(\varepsilon, \varepsilon_*) = f_\varepsilon{}'(\varepsilon, \varepsilon_*)$ are monotonically increasing functions of the arguments themeselves, $g > 0$ when $\sigma - f(\varepsilon, \varepsilon_*) > 0$; $g \equiv 0$ when $\sigma - f(\varepsilon, \varepsilon_*) \le 0$; $f(\varepsilon, \varepsilon_*)$ is the static compression diagram of the soil (when $\dot\varepsilon = \partial\varepsilon/\partial t = 0$), and ε_* is the maximum strain corresponding to the start of unloading.

From (1), we have the limiting dynamic compression diagram $\sigma = \int_0^\varepsilon E(\xi)\, d\xi \equiv \varphi(\varepsilon)$, when $\dot\varepsilon = \infty$, and the limiting static compression diagram $\sigma = f(\varepsilon, \varepsilon_*)$ when $\dot\varepsilon = 0$; for the latter diagram, there are different loading and unloading branches, as defined by the condition $\sigma - f(\varepsilon, 0) \le 0$.

The $\varphi(\varepsilon)$ and $f(\varepsilon, 0)$ functions in [1, 2] are obtained in the form

$$\varphi(\varepsilon) = E_1\left(\varepsilon + m_1\varepsilon^{\nu_1}\right),\; f(\varepsilon, 0) = K_1\left(\varepsilon + m_2\varepsilon^{\nu_2}\right)$$

(E_1, K_1, m_1, m_2, ν_1, and ν_2 are experimental coefficients given by Rykov [3] for the sands and clays with allowance for the large range of loads as compared with [1, 2]; Table 1: rows 1 and 2 correspond to a sand with a mass density $\rho_0 = 1.50\text{-}1.52$ g/cm^3, and a gravimetric moisture content $w = 0.05$ and 0.15, respectively; and, row 3 corresponds to a dense clay with $\rho_0 = 1.70$ g/cm^3 and $w = 0.22$).

The $f(\varepsilon, \varepsilon_*)$ diagram is much more precisely defined during unloading as compared with [1, 2], and considering the cyclic loading, is represented as

$$f(\varepsilon, \varepsilon_*) = (\varepsilon - \varepsilon_0)\left[K_* + (\sigma_*/(\varepsilon_* - \varepsilon_0) - K_*)\, e^{-\alpha_*(\varepsilon_* - \varepsilon_0)}\right]. \tag{2}$$

Here, ε_0 is the residual deformation corresponding to unloading of the previous load, K_* is the initial compression modulus during repeated loading (see Table 1), and α_* is a coefficient computed from the condition of continuity and smoothness of curve (2) at point $\varepsilon = \varepsilon_*$, $\sigma = \sigma_* = f(\varepsilon_*, 0)$:

$$\alpha_* = \left[\frac{(\varepsilon_* - \varepsilon_0)E_* - \sigma_*}{\sigma_* - K_*(\varepsilon_* - \varepsilon_0)}\right]\frac{1}{\varepsilon_* - \varepsilon_0},\; E_* = \partial\varphi(\varepsilon)/\partial\varepsilon\,|_{\varepsilon = \varepsilon_*}.$$

The ε_0 (ε_*) relationship for a cyclic loading is also established on the basis of the processing of experimental results [1, 2, 4] (Fig. 1):

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TABLE 1

Specimen No.	$\frac{E_1,\ \text{MPa}}{K_1,\ \text{MPa}}$	$\frac{m_1}{m_2}$	$\frac{\nu_1}{\nu_2}$	c_0, m/sec	K_*, MPa	k_*	m_*
1	$\frac{100}{15}$	$\frac{137,3}{117,6}$	$\frac{1,96}{2,37}$	250	173	0,74	0,94
2	$\frac{321,6}{30\ ^{*)}}$	$\frac{35,1}{160\ ^{*)}}$	$\frac{1,83}{2,50\ ^{*)}}$	460	153	0,70	0,00
3	$\frac{2992}{10}$	$\frac{0,6}{2600.0}$	$\frac{1,66}{3.00}$	1189	200	0,75	1,52

*Coefficients were obtained on the basis of the codification of experimental-test data [2, 3].

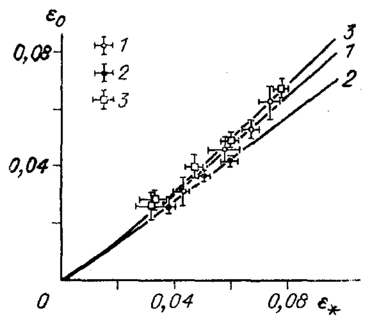

Fig. 1

$$\varepsilon_0 = k_* \left(\varepsilon_* + m_* \varepsilon_*^2\right)$$

(k_* and m_* are experimental coefficients (see Table 1)). Curves 1-3 in Fig. 1 correspond to the data of rows 1-3 in Table 1.

In contrast to [1, 2], the $g(\sigma, \varepsilon)$ function was adopted in the form

$$g = \eta(\sigma)(\sigma - f(\varepsilon, \varepsilon_*))^{\varkappa}, \tag{3}$$

$$\eta = \begin{cases} \eta_0, & \partial\sigma/\partial t \geqslant 0, \\ \eta_0 e^{-\alpha(\sigma_{**} - \sigma)}, & \partial\sigma/\partial t < 0 \end{cases}$$

for the processing of the results of cyclic dynamic tests (σ_{**} is the maximum stress attained in a given load cycle, and $\varkappa$, η_0, and α are experimental coefficients).

The method by which $\varkappa$, η_0, and α are determined is based on the selection of their combinations, which as in [1, 2], minimize the standard deviation δ of the computed $\varepsilon(t)$ curves from the average statistical curves obtained in experiment under a cyclic dynamic load:

$$\delta = \frac{\sqrt{D}}{\sum\limits_{i=1}^{s} \sum\limits_{j=j_0}^{n} \langle \varepsilon_{ij} \rangle}, \quad D = \sum_{i=1}^{s} \sum_{j=j_0}^{n} \left[\varepsilon_{ij}(\varkappa_1, \varkappa_2, \varkappa_3) - \langle \varepsilon_{ij} \rangle\right]^2, \tag{4}$$

where $\varkappa_1 = \varkappa$, $\varkappa_2 = \eta_0$, $\varkappa_3 = \alpha$ are minimization parameters, $\langle \varepsilon_{ij} \rangle = \frac{1}{m} \sum\limits_{l=1}^{m} \varepsilon_{il}(t_j)$ is the average strain value when $t = t_j$ from the results of a series of tests for a certain deformation regime, m is the number of tests in the series, j_0 is the number of the interval corresponding to the moment when a quasi-static regime is established [2], s is the number of load cycles on the same specimen in the series, $\varepsilon_{ij} \equiv \varepsilon_i(t_j)$ is the computed strain value for given times $t = t_j$ ($j = 1, 2, \ldots, n$), and n is the number of intervals into which time is partitioned in processing the experimental results.

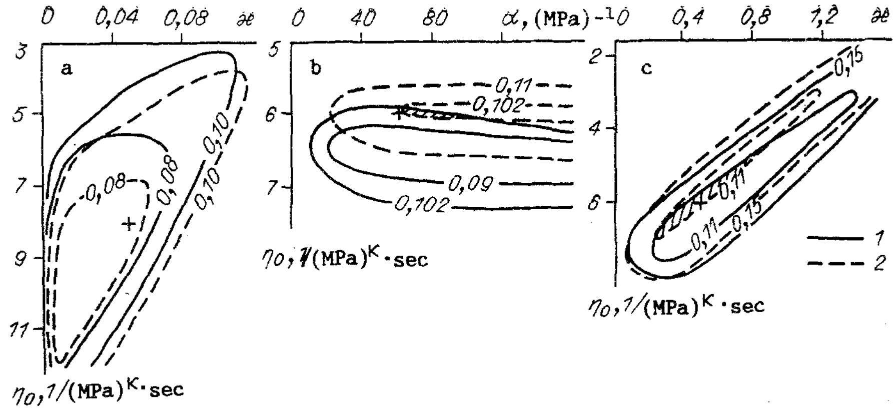

Fig. 2

The computed $\varepsilon(t)$ curve is calculated from (1) by integrating for a load $\sigma(t)$ known from experiment:

$$\begin{gathered}\varepsilon_{j+1} = \varepsilon_j + \Delta\varepsilon_{j+1/2},\\ \Delta\varepsilon_{j+1/2} = \frac{1}{E\left(\tilde{\varepsilon}_{j+1/2}, \varepsilon_*\right)}\Delta\sigma_{j+1/2} + \eta\left[\sigma_{j+1/2} - f\left(\tilde{\varepsilon}_{j+1/2}, \varepsilon_*\right)\right]^{\varkappa}\Delta t,\\ \Delta\sigma_{j+1/2} = \sigma_{j+1} - \sigma_j,\ \sigma_{j+1/2} = (\sigma_j + \sigma_{j+1})/2\\ (\varepsilon_j \equiv \varepsilon(t_j),\ \tilde{\varepsilon}_{j+1/2} \equiv \tilde{\varepsilon}(t_j + (1/2)\Delta t),\ \sigma_{j+1/2} \equiv \sigma(t_j + (1/2)\Delta t)).\end{gathered} \quad (5)$$

In connection with the fact that Eq. (1) cannot be written in explicit form with respect to ε, the implicit quantity $\tilde{\varepsilon}_{j+1/2}$, the value of which is found by iteration until its deviation from the preceding is no less than 10^{-5} (the initial value $\tilde{\varepsilon}_{j+1/2} = \varepsilon_j$), is introduced to relationship (5). The partitioning increment Δt employed for the experiments and the processing of the results using the automated system in [4] was $0.2 \cdot 10^{-3}$ sec. In processing the data in [1, 2], the increment was appreciably larger. In these cases, therefore, the intermediate values of the load $\sigma(t)$ were determined using a Lagrange interpolation equation. It was assumed that those values of $\varkappa$, η_0, and α for which the condition $\delta \le \delta_0$ (δ_0 is the average (with respect to the entire process) relative confidence interval for the given series of experiments (experiment accuracy)) are the desired values.

The search for $\varkappa$, η_0, and α was carried out in the following order. The coefficients η_0 and $\varkappa$ for the loading process when $\partial\sigma/\partial t \ge 0$ ($\alpha = 0$ in accordance with (3)) were determined initially on the basis of the minimization of (4). The contour lines characterizing the distribution of the error of (4) and the plane of the coefficients $\varkappa$ and η_0 are the result of computation in this stage of calculations. Each of the effects is treated as independent, and the values of ε_0 are assigned from tests, while the dispersions are summed. In the second stage of the calculations, function (4) is minimized with respect to the parameters η_0 and α for a fixed $\varkappa$, which is adopted on the basis of the previous stage. In this case, α is determined and η_0 is defined more precisely. In the third stage, repeated minimization of function (4) with respect to $\varkappa$ and η_0 is carried out for a selected α; in this case, each succeeding cycle is treated as a continuation of the previous one, while the initial deformation is calculated for each successive cycle.

The results of this kind of calculation for two series of tests of a sandy soil with ρ_0 = 1.50-1.52 g/cm³ and w = 0.05 from [1, 2] (curves 1) and from [4] (curves 2) are presented in Fig. 2a-c. The figures on the contour lines correspond to the deviations of (4). The experimental accuracy in the tests was δ_0 = 0.10-0.12.

TABLE 2

Specimen No.	η_0, $\frac{1}{(\text{MPa})^{\varkappa}\cdot\text{sec}}$	$\varkappa$	α, $(\text{MPa})^{-1}$	δ	δ_0
1	6,0	0,5	60	0,11	0,12
2	8,0	0,5	20	0,08	0,10
3	5.0	0.5	5	0,10	0,10

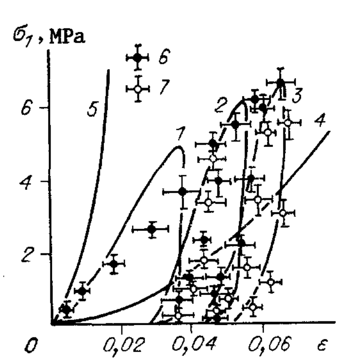

Fig. 3

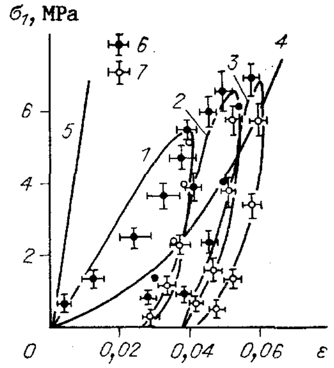

Fig. 4

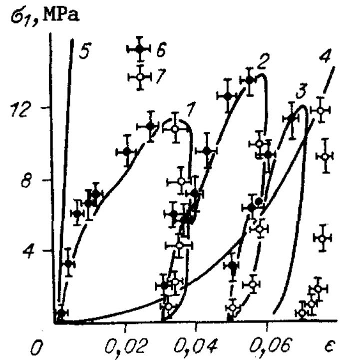

Fig. 5

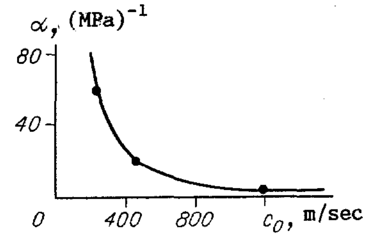

Fig. 6

It is apparent in Fig. 2a (first stage of minimization) that $\varkappa$ can be set equal to 0.5. The computational results obtained in the second stage of minimization (Fig. 2b) suggest that the minimum error δ corresponds to $\alpha = 60$ when $\eta_0 = 6.0$. It follows from the third stage of minimization (Fig. 2c) that $\varkappa = 0.5$ and $\eta_0 = 6.0$ can be taken as the computed values of $\varkappa$ and η_0.

The mechanical characteristics of the sandy soils and clays under a cyclic dynamic loading, which were obtained by this method using published data on the limiting diagrams [3], are presented in Table 2.

The results of comparison between the computed $\sigma(\varepsilon)$ curves and experimental values from [1-4] are presented in Figs. 3-5. $w = 0.05$ (Fig. 3) and $w = 0.15$ (Fig. 4) for the sand with $\rho_0 = 1.50$-1.52 g/cm^3, and $w = 0.22$ (Fig. 5) for the dense clay with $\rho_0 = 1.70$. Lines 1-3 in Figs. 3-5 were computed for three successive specimen loadings, lines 4 for the static $f(\varepsilon, 0)$ compression diagram, and lines 5 for the limiting $\varphi(\varepsilon)$ dynamic diagram. Points 6 and 7 correspond to average statistical experimental values with confidence intervals determined with a reliability of 0.95 (6 – loading, 7 – unloading in each of the cycles). It is apparent that in the majority of cases, the computed $\sigma(\varepsilon)$ curves fall within the bounds of the confidence intervals. On average, the accuracy of approximation is 8-11% (see Table 2). A more significant discrepancy occurs only during unloading in the third loading cycle of the dense-clay specimens (see Fig. 5). This is associated with the insufficient accuracy of determination of the static diagram for this soil when $\varepsilon \geq 0.06$, which in this region of deformations, is adopted for calculations on the basis of the extrapolation of experimental results [3] actually obtained in the $0 \leq \varepsilon \leq 0.06$ region.

In conclusion, let us point out that there is a certain law governing the variation in the coefficient α as a function of the spread velocity c_0 of weak disturbances in the soils under consideration (Fig. 6).

Thus, the cited strain law, which takes into account the variation in the viscous properties of sandy and clayey soils during unloading, makes it possible to describe the cyclic dynamic loading of soils in uniaxial compression under plane strain with a sufficient degree of accuracy. In this case, the results of determination of the coefficients $\varkappa$ and η_0

differ from those presented in [1, 2]; this is associated with both the more precise definition of data on the limiting dynamic compression diagrams of the soils under consideration, and allowance for the peculiarities of their cyclic loading.

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METHOD OF ELASTIC CHARACTERISTIC VARIATION IN THE PROBLEM OF A LIMITING LOAD

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One of the questions of ideal plasticity theory is that of finding limiting loads with which a structure ceases to resist the action of external forces. Two-way evaluation of them may be obtained with the help of static and kinematic theorems [1]. Given below is a procedure based on these theorems making it possible to approach successively the upper and lower boundaries of the limiting load.

Let the condition for yielding have the form (the Mises–Hill criterion)

$$I = \sigma^{T} A \sigma = 1, \tag{1}$$

where σ is a vector-column composed of stress tensor components; A is a matrix of plastic flow characteristics; symbol T means the operation of transposing.

Equations for equilibrium within the body and at its boundary are written in operator form

$$D\sigma(x) = q(x),\ q(x) = q_0(x)t. \tag{2}$$

Here D is a matrix of linear differential operators; $q_0(x)$ is normalized external load; t is loading parameter; x is radius-vector for a point of the body.

Coefficient t_* is sought on reaching which the structure loses its supporting capacity.

Lower Estimate. The solution of Eq. (2) is presented in the following symbolic form: $\sigma = \sigma_0 t$, $\sigma_0 = D^{-1} q_0$. We calculate function I: $I = I_0 t^2$, $I_0 = \sigma_0^T A \sigma_0$. Let with $t = t_-$ stress σ be reached for the flow surface of any point of the body. Then

$$(I_0)_{\max_x} t_-^2 = 1.$$

Since equilibrium equations are satisfied and stresses do not go beyond flow surface (1), then according to the static theorem

$$t_* \geqslant t_- = 1 \Big/ \sqrt{(I_0)_{\max_x}}. \tag{3}$$

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